Using complete sentences and proper mathematical notation, state the **formal definition** of "concave up". **SCORE: _____ / 2 PTS**

f IS CONCAVE UP ON $[a,b]$ IF AND ONLY IF f' IS INCREASING ON $[a,b]$

Using complete sentences and proper mathematical notation, state the **formal definition** of "critical number". **SCORE: _____ / 2 PTS**

c IS A CRITICAL NUMBER OF f IF AND ONLY IF c IS IN THE DOMAIN OF f AND $f'(c)=0$ OR IS UNDEFINED

Find the global extrema of $f(x) = x^{\frac{2}{3}}(x-25)$ on the interval $[-1, 8]$. **SCORE: _____ / 6 PTS**

$f(x) = x^{\frac{5}{3}} - 25x^{\frac{2}{3}}$

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{50}{3}x^{-\frac{1}{3}}$ IS UNDEFINED AT $x=0$ (1)

(1½) $= \frac{5}{3}x^{-\frac{1}{3}}(x-10) = 0$ AT (1) $x = 10 \notin [-1,8]$

(−1) IF YOU FOUND $f(10)$

$f(-1) = (-1)^{2/3}(-1-25) = 1(-26) = -26$ (½)

$f(0) = 0^{2/3}(0-25) = 0(-25) = 0$ (½) ⟵ MAX (½)

$f(8) = 8^{2/3}(8-25) = 4(-17) = -68$ (½) ⟵ MIN (½)

Sketch the graph of a **continuous** function $f(x)$ that satisfies all the following conditions. **SCORE: _____ / 4 PTS**

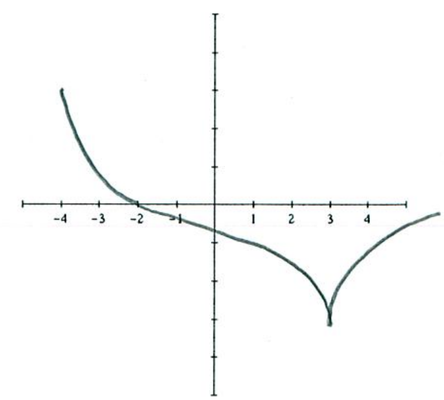
$f'(-2) = 0$

$f'(x) < 0$ if $x < -2$ or $-2 < x < 3$, and $f'(x) > 0$ if $x > 3$

$f''(x) > 0$ if $x < -2$, and $f''(x) < 0$ if $-2 < x < 3$ or $x > 3$

f' − 0 − +

f'' + − −



$f(x)$ is a polynomial function such that $f'(-6) = f'(8) = 0$ and $f''(x) = (-16 - 5x)(8 - x)^3$. **SCORE: _____ / 3 PTS**

For each critical number of f, determine what the Second Derivative Test tells you about that critical number.
Justify your answer very briefly.

WRONG IF YOU SAID "NOT AN EXTREMA"

$f''(-6) = (-16 - -30)(8 - -6)^3 > 0$ $\frac{1}{2}$ → LOCAL MIN 1

$f''(8) = (-16 - 40)(8 - 8)^3 = 0$ $\frac{1}{2}$ → NO CONCLUSION 1

$f(x)$ is a polynomial function with derivative $f'(x) = (5 + x)^4 (7 - x)$. **SCORE: _____ / 5 PTS**

[a] Find the critical numbers of f. **Justify your answer very briefly.**

$\frac{1}{2}$ $f'(x) = 0$ AT $x = -5, 7$ 1

[b] For each critical number of f, determine what the First Derivative Test tells you about that critical number.
Justify your answer very briefly. $1\frac{1}{2}$

		-5		7	
f'	+		+		−
$(5+x)^4$	+		+		+
$(7-x)$	+		+		−

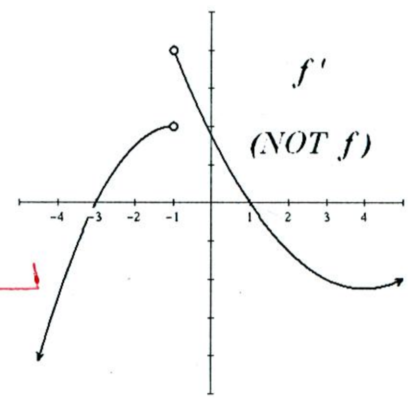
$x = -5$ IS NOT AN EXTREMA 1

$x = 7$ IS A LOCAL MAX 1

$f(x)$ is a continuous function whose derivative $f'(x)$ is shown on the right. **SCORE: _____ / 4 PTS**

The following questions are about the function f, **NOT THE FUNCTION f'**.

[a] Write "I UNDERSTAND" if you understand that the following questions are about the continuous function f, **NOT THE FUNCTION f'**.



[b] Find the critical numbers of f.
Justify your answer very briefly.

1 $f'(x)$ DNE AT $x = -1$

1 $f'(x) = 0$ AT $x = -3, 1$

[c] Find the x – coordinates of all local maxima of f.
Justify your answer very briefly.

1 f' CHANGES FROM + TO − AT $x = 1$

[d] Find all intervals over which f is concave down.
Justify your answer very briefly.

1 f' IS DECREASING ON $(-1, 4]$

Let $f(x)$ be a function such that $f(1) = 3$ and $f'(x) < 2$ for all $x \in [1, 5]$. **SCORE: _____ / 4 PTS**

Prove that $f(5) < 13$. **HINT: Write a proof by contradiction as shown in lecture.**

ASSUME $f(5) \geq 13$ 1

SINCE f' EXISTS ON $[1,5]$, f IS DIFFERENTIABLE + CONTINUOUS ON $[1,5]$ $\frac{1}{2}$

$\frac{1}{2}$ BY MVT, $f'(c) = \dfrac{f(5) - f(1)}{5 - 1} \geq \dfrac{13 - 3}{4} = 2\frac{1}{2}$ FOR SOME $c \in [1,5]$ 1

BUT $f'(x) < 2$ ON $[1,5]$, SO BY CONTRADICTION, $f(5) < 13$. 1